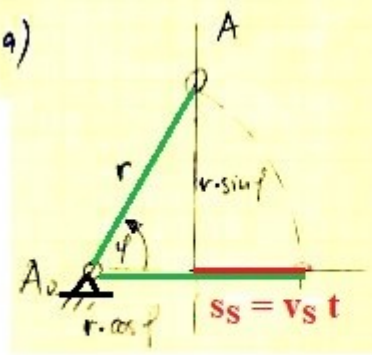
Musterlösung 2. Probeklausur

1. a)



$$\cos\varphi = \frac{r - v_s \cdot t}{r}$$

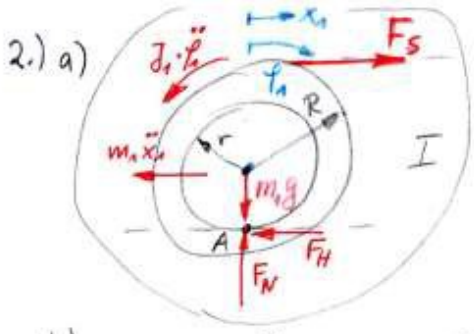
$$\cos\varphi = 1 - \frac{v_s \cdot t}{r}$$

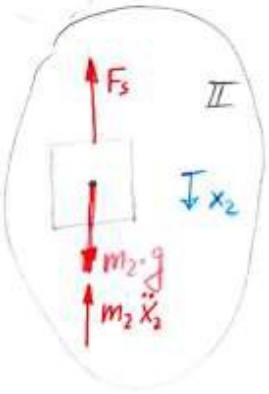
$$\varphi = \arccos\left(1 - \frac{v_s \cdot t}{r}\right)$$

$$\omega_A = \frac{d\varphi}{dt} = -\frac{-\frac{v_s}{r}}{\sqrt{1-\left(1-\frac{v_s \cdot t}{r}\right)^2}}$$

$$\omega_A = \frac{v_s}{r}\left[1-\left(1-\frac{v_s \cdot t}{r}\right)^2\right]^{-\frac{1}{2}}$$

2.) a)





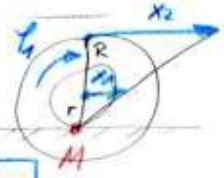
b)

$$\text{I}: \curvearrowleft (A): J_1 \ddot{\varphi}_1 + m_1 \ddot{x}_1 \cdot r - F_S \cdot (R+r) = 0$$

$$\text{II}: \uparrow: F_S + m_2 \ddot{x}_2 - m_2 g = 0$$

$$F_S = m_2 g - m_2 \ddot{x}_2$$

$$J_1 \ddot{\varphi}_1 + m_1 \ddot{x}_1 \cdot r - (m_2 \cdot g - m_2 \ddot{x}_2)(R+r) = 0$$



Zwangsbed.: $\frac{x_1}{r} = \frac{x_2}{R+r} \Rightarrow x_1 = x_2 \cdot \frac{r}{R+r}$

$$\ddot{x}_1 = \ddot{x}_2 \frac{r}{R+r}$$

Rollbed.: $x_1 = r \cdot \varphi_1$

$\ddot{x}_1 = r \cdot \ddot{\varphi}_1 \Rightarrow$

$$\ddot{\varphi}_1 = \frac{\ddot{x}_1}{r} = \frac{\ddot{x}_2}{R+r}$$

$J_1 = 4 m_1 r^2$ (vorgegeben)

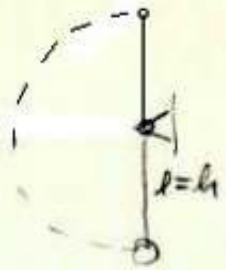
$$4 m_1 r^2 \cdot \frac{\ddot{x}_2}{R+r} + m_1 \cdot \ddot{x}_2 \frac{r}{R+r} \cdot r - m_2 g (R+r) + m_2 \ddot{x}_2 (R+r) = 0$$

$$\ddot{x}_2 \left[\frac{4 m_1 r^2}{R+r} + \frac{m_1 r^2}{R+r} + m_2 (R+r)\right] = m_2 g (R+r) \quad | \cdot (R+r)$$

$$\ddot{x}_2 \left[5 m_1 r^2 + m_2 (R+r)^2\right] = m_2 g (R+r)^2$$

$$\ddot{x}_2 = \frac{m_2 g \cdot (R+r)^2}{\left[5 m_1 r^2 + m_2 (R+r)^2\right]} = \frac{10\,\text{kg} \cdot 9{,}81 \frac{\text{m}}{\text{s}^2} (0{,}5)^2 \text{m}^2}{\left[5 \cdot 25\,\text{kg} \cdot 0{,}2^2 \text{m}^2 + 10\,\text{kg} \cdot 0{,}5^2 \text{m}^2\right]}$$

$$\ddot{x}_2 = 3{,}27 \frac{\text{m}}{\text{s}^2}$$

3 a) 

$v_1 = \sqrt{2g \cdot 2l} = 4{,}43 \frac{m}{s}$

b) $m_1 \cdot v_1 = m_1 \cdot u_1 + m_2 \cdot u_2$ (2)

$k = \frac{u_1 - u_2}{-v_1} \Rightarrow u_2 = k \cdot v_1 + u_1$ (1)

(2) in (1): $m_1 \cdot v_1 = m_1 \cdot u_1 + m_2 (k \cdot v_1 + u_1)$

$m_1 \cdot v_1 = m_1 u_1 + m_2 \cdot k \cdot v_1 + m_2 \cdot u_1$

$u_1 (m_1 + m_2) = m_1 v_1 - m_2 \cdot k \cdot v_1$

$u_1 = v_1 \cdot \frac{m_1 - k \cdot m_2}{m_1 + m_2} = \frac{(0{,}5 - 0{,}9 \cdot 2)}{2{,}5} 4{,}43 = -2{,}30$ m/s

$u_2 = 0{,}9 \cdot 4{,}43 - 2{,}3 = 1{,}69$ m/s

c) $W_{kin,1} = W_{pot}$

$\frac{1}{2} m_2 \cdot u_2^2 = m_2 g \cdot h_1 \Rightarrow h_1 = \frac{u_2^2}{2g} = 0{,}145$ m

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4.a) $m_{ges} = m_{Zyl.} + 3 \cdot m_{Speiche} + m_{Ring}$

$m_{Zyl} = \varrho \cdot \frac{\pi \cdot 0{,}15^2}{4} \cdot 0{,}15 = 0{,}021$ kg

$m_{Sp.} = 0{,}2 \cdot 0{,}15 \cdot 0{,}08 \cdot \varrho = 0{,}019$ kg

$m_{Ring} = \varrho \cdot \frac{\pi}{4} (0{,}75^2 - 0{,}55^2) \cdot 0{,}15 =$

$0{,}52 - 0{,}28 = 0{,}24$ kg

$m_{ges} = 0{,}021 + 3 \cdot 0{,}019 + 0{,}24 = 0{,}32$ kg

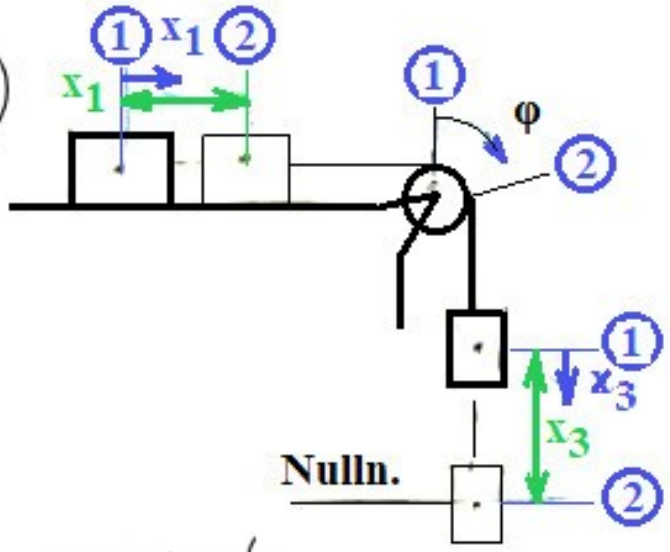
b) $J_{ges} = J_{Zyl.} + 3 (J_{Sp} + m_{Sp} r_{Sp}^2) + J_{Ring}$

$= \frac{1}{2} m_{Zyl.} \cdot 7{,}5^2 + 3 (\frac{1}{12} m_{Sp.} 20^2 + m_{Sp.} \cdot 17{,}5^2) + \frac{1}{2} \cdot 0{,}52 \cdot 32{,}5^2 - \frac{1}{2} 0{,}28 \cdot 27{,}5^2$

$= 0{,}59 + 3 (0{,}63 + 5{,}82) + 365{,}63 - 105{,}88$ kgmm^2

$J_{ges} = 280$ kg mm^2

5)



Energiesatz

$$W_{pot(m_3)} - W_{ab} = W_{kin,tr.(m_1)} + W_{kin,rot(m_2)} + W_{kin,tr(m_3)}$$

$$m_3 \cdot g \cdot x_3 - m_1 \cdot g \cdot \mu \cdot x_1 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} J_2 \cdot \omega_2^2 + \frac{1}{2} m_3 \cdot v_3^2$$

Zwangsbed.

$$x_1 = x_3 \; ; \; v_1 = v_3$$

$$v_3 = \omega_2 \cdot r \quad \Rightarrow \omega_2 = \frac{v_3}{r}$$

Massenträgh. $J_2 = \frac{1}{2} m_2 r_2^2$

$$m_3 \cdot g \cdot x_3 - m_1 \cdot g \cdot \mu \cdot x_3 = \frac{1}{2} m_1 \cdot v_3^2 + \frac{1}{2} m_3 \cdot v_3^2 + \frac{1}{2} m_2 r^2 \cdot \frac{v_3^2}{r^2} \cdot \frac{1}{2}$$

$$= v_3^2 \left(\frac{m_1}{2} + \frac{m_3}{2} + \frac{m_2}{4} \right)$$

$$v_3 = \sqrt{\frac{g \cdot x_3 (m_3 - m_1 \cdot \mu)}{\frac{1}{2}(m_1 + \frac{m_2}{2} + m_3)}} = \sqrt{\frac{9{,}81 \cdot 0{,}5 \,(5 - 10 \cdot 0{,}15)}{0{,}5 \,(16)}}$$

$$v_3 = \sqrt{\frac{17{,}17}{8}} = 1{,}46 \; \frac{m}{s}$$